

(i) Printed Pages: 3

Roll No. ....

(ii) Questions : 8

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B.A./B.Sc. (General) 5<sup>th</sup> Semester  
(2122)

MATHEMATICS

Paper-I : Analysis-I

Time Allowed : Three Hours]

[Maximum Marks : 30

Note :—Attempt five questions in all, selecting at least two questions from each Unit.

UNIT-I

1. (a) Show that set of integers is countable.

(b) Let  $f(x) = \frac{1}{x^2}$  on  $[1, 4]$ . Find  $L(p, f)$  by dividing  $[1, 4]$  into three equal sub-intervals. 3+3=6

2. (a) Prove that every continuous function on closed interval is Riemann integrable.

(b) If  $f$  is R-integrable on  $[a, b]$  and  $|f(x)| \leq k \forall x \in [a, b]$  then prove that :

$$\left| \int_a^b f(x) dx \right| \leq k(b-a), \quad 3+3=6$$

3. (a) Show that  $\frac{1}{3\sqrt{2}} \leq \int_0^1 \frac{x^2}{\sqrt{1+x^2}} dx \leq \frac{1}{3}$

(b) Give an example of a bounded function "f" defined on a closed interval such that |f| is R-integrable but f is not. 3+3=6

4. (a) Prove that  $\int_{-1}^{\infty} \frac{x+1}{(x+2)^6} dx = \frac{1}{20}$ .

(b) Use  $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$  to prove  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ .

3+3=6

### UNIT-II

5. (a) Test for convergence of integral  $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$ .

(b) Examine for convergence  $\int_0^{\infty} \left(\frac{1}{x} - \frac{1}{\sinh x}\right) \frac{dx}{x}$ . 3+3=6

6. (a) Use Dirichlet's test to show  $\int_0^{\infty} \frac{\sin x}{x} dx$  is convergent at  $\infty$ .

(b) State and prove Abel's test for convergence of Improper integral. 3+3=6

7. (a) Discuss the convergence of the integral

$$\int_1^2 \frac{dx}{(x-1)^{1/2} (2-x)^{1/3}}$$

- (b) Use Frullani Theorem and prove that :

$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} dx = \log\left(\frac{b}{a}\right) \text{ where } a > 0, b > 0.$$

3+3=6

8. (a) If  $|a| < 1$  then evaluate  $\int_0^{\pi} \frac{\log(1 + a \cos x)}{\cos x} dx$ .

- (b) Using  $\int_0^{\infty} \frac{dx}{x^2 + a} = \frac{\pi}{2\sqrt{a}}$ , prove that :

$$\int_0^{\infty} \frac{dx}{(x^2 + a)^{n+1}} = \frac{\pi}{2} \frac{1.3.5 \dots (2n-1)}{2.4.6 \dots (2n)} \cdot \frac{1}{a^{n+\frac{1}{2}}}$$

3+3=6

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